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Computers and Mathematics with Applications 56 (2008) 813–821

An International Journal
computers & mathematics with applications

www.elsevier.com/locate/camwa

Numerical solution of linear and nonlinear Black–Scholes option pricing equations

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Abstract

This paper deals with the numerical solution of Black–Scholes option pricing partial differential equations by means of semidiscretization technique. For the linear case a fourth-order discretization with respect to the underlying asset variable allows a highly accurate approximation of the solution. For the nonlinear case of interest modeling option pricing with transaction costs, semidiscretization technique provides a competitive numerical solution with respect to others recently given in [B. Düring, M. Fournier, A. Jüngel, Convergence of a high order compact finite difference scheme for a nonlinear Black–Scholes equation, Esaim–Math. Modelling Numer. Anal.–Modelisation Mathematique et Analyse Numerique 38 (2004) 359–369; B. Düring, Black–Scholes type equations: mathematical analysis, parameter identification & numerical solution, Dissertation, University Mainz, July 2005].

Keywords: Black-Scholes equation; Numerical solution; Option pricing; Semidiscretization; Transaction cost

1. Introduction

Although the Black-Scholes (B-S) model has been widely accepted by academics and used by practitioners, it has also attracted criticism because the essential model parameter, the volatility, is not observable. It is often determined by computing the so-called implied volatility out of the observed option prices by inverting the B-S formula. A widely observed phenomenon – the so-called smile/skew effect – is that, in contradiction to the B-S model assumptions, these computed volatilities are not constant. This leads to a natural generalization of the B-S model replacing the constant volatility σ in the model by a local volatility function $\sigma = \sigma(E, T)$ where E denotes the exercise price.

In practice, transaction costs arise when trading securities. Although they are generally small for institutional investors, recent studies of their influence show that they lead to a notable increase in the option price. In recent years, different models have been proposed to weaken unrealistic assumptions of the B-S model. These models result in strongly of fully nonlinear B-S equations in order to model.

- transaction costs arising in the hedging of portfolios [1,3–5]
- feedback effects due to large traders [6–11]
- incomplete markets

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doi:10.1016/j.camwa.2008.02.010

In this paper we are interested in the option pricing with transaction costs. In 1992, Boyle and Vorst [4] derived from a binomial model an option price taking into account transaction costs and that is equal to a B-S price but with a modified volatility of the form

$$\sigma = \sigma_0 (1 + cA)^{1/2}, \quad A = \frac{\mu}{\sigma_0 \sqrt{\Delta T}}, \quad c = 1.$$

Here, μ is the proportional transaction cost, Δt the transaction period, and σ_0 is the original volatility constant. Leland [12] computed $c = (\frac{2}{\pi})^{1/2}$. Kusuoka [13] then showed that the "optimal" c depends on the risk structure of the market. Pars and Avellaneda [14] derived the modified volatility,

$$\sigma = \sigma_0 (1 + A \operatorname{sign}(V_{SS}))^{1/2},$$

from a binomial model using the algorithm of Bensaid et al. [15]. Here, V is the option price, S the price of the underlying asset and V_{SS} denotes the second derivative of V with respect to S (the "Gamma"). In particular, the option price does not need to be convex.

A more complex model has been proposed by Barles and Soner [3]. In their model the nonlinear volatility reads

$$\sigma^2 = \sigma_0^2 (1 + \Psi[\exp(r(T-t)a^2 S^2 V_{SS})]), \tag{1}$$

where r is the risk-free interest rate, T the maturity and $a = \mu\sqrt{\gamma N}$ in which γ is the risk aversion factor and N is the number of options to be sold. The function Ψ is the solution of the nonlinear initial-value problem

$$\Psi'(A) = \frac{\Psi(A) + 1}{2\sqrt{A\Psi(A)} - A}, \qquad A \neq 0, \quad \Psi(0) = 0. \tag{2}$$

In the mathematical literature, only a few results can be found on the numerical discretization of B-S equation, mainly for linear B-S equations. The numerical approaches vary from finite element discretizations [16,17] to finite difference approximations [18]. The numerical discretization of the B-S equations with the nonlinear volatility (2) has been performed using explicit finite-difference schemes [3,14]. However, explicit schemes have the disadvantage that restrictive conditions on the discretization parameters (for instance, the ratio of the time and the space step) are needed to obtain stable, convergent schemes [19]. Moreover, the convergence order is only one in time and two in space. [1,2] combines high-order compact difference schemes derived by [20] and techniques to construct numerical solutions with frozen values of the nonlinear coefficient of the nonlinear B-S equation to make the formulation linear.

In this paper we use a semidiscretization technique by using fourth-order difference approximations of the partial derivatives V_S *and* V_{SS} arising in the nonlinear B-S equation

$$V_t + \frac{1}{2}\sigma(V_{SS})^2 S^2 V_{SS} + rSV_S - rV = 0. \tag{3}$$

Then we achieve an ordinary system of nonlinear ordinary differential equations with respect to the time, that is solved numerically. Apart form (3), in the Barles-Soner model one has the terminal condition

$$V(S,T) = \max(0, S - E), \quad S > 0, \tag{4}$$

and the boundary conditions

$$V(0,t) = 0, \qquad \lim_{s\to\infty} \frac{V(S,t)}{S - E\mathrm{e}^{-r(T-t)}} = 1. \tag{5}$$

In [1], after using variable transformations, one translates the asymptotic boundary condition to the boundary of the bounded limited domain when the numerical solution is constructed.

In this paper we semidiscretize (3) with respect to the variable S regarding a bounded interval $[E-L, E+L]$, where E is the strike price and L is a suitable value such that $E - L < 0$.

This paper is organized as follows. Section 2 deals with some preliminaries about the semidiscretization technique as well as an implicit expression of the solution of Eq. (2). In Section 3 the linear case is treated with the advantage that the resulting ordinary system of differential equations can be solved in closed form. An illustrative example is included. Finally in Section 4 the nonlinear case is addressed using semidiscretization technique and numerical results are compared with those of [1].

2. Preliminaries

We begin this section by introducing the semidiscretization technique to equation

$$U_\tau - \frac{S^2}{2}\sigma^2 U_{SS} - rSU_S + rU = 0, \quad 0 < S < \infty, 0 < \tau \le T, \tag{6}$$

resulting from [3] after applying the change of variable $\tau = T - t$. Note that terminal condition (4) takes the form

$$U(S, 0) = \max(0, S - E), \tag{7}$$

and the boundary conditions

$$U(0, \tau) = 0, \quad \lim_{S\to\infty} \frac{U(S, \tau)}{S - E\mathrm{e}^{-r\tau}} = 1. \tag{8}$$

Let us approximate the S derivatives of U at (S, τ) by

$$\frac{\partial U}{\partial S}(S, \tau) = \frac{U(S - 2h, \tau) - 8U(S - h, \tau) + 8U(S + h, \tau) - U(S + 2h, \tau)}{12h} + O(h^4), \tag{9}$$

$$\frac{\partial^2 U}{\partial S^2}(S, \tau) = \frac{-U(S - 2h, \tau) + 16U(S - h, \tau) - 30U(S, \tau) + 16U(S + h, \tau) - U(S + 2h, \tau)}{12h^2} + O(h^4), \tag{10}$$

see [21–23].

Let us subdivide the underlying asset variable interval $[E - L, E + L]$ into N equal subintervals by the grid lines $S_i = E - L + ih, 0 \le i \le N$ where $Nh = 2L$, and write down Eqs. (9) and (10) at every mesh point S_i along time-level τ. It then follows that the values $u_i(\tau)$ approximating $U(S_i, \tau)$ will satisfy the system of ordinary differential equations

$$\frac{\mathrm{d}u(\tau)}{\mathrm{d}\tau} = Bu(\tau) + w, \tag{11}$$

where $u(\tau) = [u_1, u_2, \ldots, u_{N-1}]^T$, w is a column vector, with

$$B = \begin{bmatrix} \gamma_1 & \delta_1 & \xi_1 & 0 & 0 & 0 & 0 & \ldots & 0 \\ \beta_2 & \gamma_2 & \delta_2 & \xi_2 & 0 & 0 & 0 & \ldots & 0 \\ \alpha_3 & \beta_3 & \gamma_3 & \delta_3 & \xi_3 & 0 & 0 & \ldots & 0 \\ 0 & \alpha_4 & \beta_4 & \gamma_4 & \delta_4 & \xi_4 & 0 & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & \ldots & 0 & 0 & \alpha_{N-3} & \beta_{N-3} & \gamma_{N-3} & \delta_{N-3} & \xi_{N-3} \\ 0 & \ldots & 0 & 0 & 0 & \alpha_{N-2} & \beta_{N-2} & \gamma_{N-2} & \delta_{N-2} \\ 0 & \ldots & 0 & 0 & 0 & 0 & \alpha_{N-1} & \beta_{N-1} & \gamma_{N-1} \end{bmatrix}, \tag{12}$$

and

$$w = \begin{bmatrix} \alpha_1 u_{-1} + \beta_1 u_0 \\ \alpha_2 u_0 \\ 0 \\ \vdots \\ 0 \\ \xi_{N-2} u_N \\ \delta_{N-1} u_N + \xi_{N-1} u_{N+1} \end{bmatrix}, \tag{13}$$

where

$$
\begin{aligned}
\alpha_i &= \alpha_i(\tau) = -\frac{\sigma_i^2 S_i^2}{24h^2} + \frac{rS_i}{12h},\\
\beta_i &= \beta_i(\tau) = \frac{2\sigma_i^2 S_i^2}{3h^2} - \frac{2rS_i}{3h},\\
\gamma_i &= \gamma_i(\tau) = -\frac{15\sigma_i^2 S_i^2}{12h^2} - r,\\
\delta_i &= \delta_i(\tau) = \frac{2\sigma_i^2 S_i^2}{3h^2} + \frac{2rS_i}{3h},\\
\xi_i &= \xi_i(\tau) = -\frac{\sigma_i^2 S_i^2}{24h^2} - \frac{rS_i}{12h},
\end{aligned}
\tag{14}
$$

where $\sigma_i = \sigma(U_{SS}(S_i, \tau))$ according to (1). Note that from (2) one gets $\sigma_i = \sigma_0$ for the linear case, i.e., no transaction cost, a=0, $\Psi(0) = 0$.

As we noted above the values of $\Psi(A)$ play an important role in the solution of (3) throughout (1). After [3], the numerical solution of (3)–(5) has always been based on the numerical solution of (2). Even in the case where the numerical method used to solve (2) needs to be accurate, the numerical error in the evaluation of $\Psi(A)$ should be reduced because from (1) it also involves approximations of U_{SS} and the global numerical error can be accumulated dramatically.

This fact motivates the search for the exact solution of the nonlinear volatility correction function Ψ – a unique solution of (2), see Theorem 3.1 of [3]. In order to solve (2), we distinguish two subdomains to the right and the left of the origin $A = 0$, where the initial condition $\Psi(0) = 0$ is given.

Case 1: $A > 0$

Let us consider the change of variables defined by

$$A = z^2, \qquad g(\Psi) = \sqrt{\Psi}, \tag{15}$$

straightforward computations show that using (15) Eq. (2) is transformed into

$$(1 + g^2)\frac{dz}{dg} = 2g^2 - gz. \tag{16}$$

From 1.1.4. of [24] one gets

$$z = \exp\left(-\frac{1}{2}\ln(g^2 + 1)\right)\left[C + \int \frac{2g^2}{\sqrt{g^2+1}}dg\right],$$

and by integrating it follows that

$$z = \frac{C - \operatorname{arcsinh} g}{\sqrt{1 + g^2}} + g\,. \tag{17}$$

As $\Psi(0)$, from (15), $z = 0$, $g = 0$ and from (17), $C = 0$.

Thus the solution of (2) for $A > 0$ satisfies

$$\sqrt{A} = -\frac{\operatorname{arcsinh}\sqrt{\Psi}}{\sqrt{\Psi + 1}} + \sqrt{\Psi}. \tag{18}$$

Case 2: $A < 0$ Considering the change of variables defined by

$$A = -z^2, \quad g(\Psi) = \sqrt{-\Psi}, \tag{19}$$

computing and taking into account (19), Eq. (2) becomes

$$(1 - g^2)\frac{dz}{dg} = 2g^2 + gz. \tag{20}$$

From 1.1.4 of [24] one gets

$$z = \frac{C + \arcsin g}{\sqrt{1-g^2}} - g. \tag{21}$$

Taking into account (19) and the initial condition $\Psi(0) = 0$, it follows that

$$\sqrt{-A} = \frac{\arcsin \sqrt{-\Psi}}{\sqrt{\Psi + 1}} - \sqrt{-\Psi}, \quad A < 0, \quad -1 < \Psi < 0. \tag{22}$$

Now we use the analytic expressions of Ψ in order to obtain a qualitative property of the nonlinear volatility correction Ψ, in fact Ψ is an increasing function.

Case 1: $A > 0$, $\Psi > 0$

Note that from (2) one get

$$\operatorname{sign}\left(\frac{\mathrm{d}\Psi}{\mathrm{d}A}\right) = \operatorname{sign}\,(2\sqrt{A\Psi} - A) \; = \operatorname{sign}\,(2\sqrt{\Psi} - \sqrt{A}). \tag{23}$$

Taking into account (18) it follows that

$$2\sqrt{\Psi} - \sqrt{A} = \sqrt{A} + \frac{2\,\operatorname{arcsinh}\sqrt{\Psi}}{\sqrt{\Psi+1}} > 0. \tag{24}$$

From (23) and (24) one gets that $\frac{\mathrm{d}\Psi}{\mathrm{d}A} > 0$ and thus Ψ is an increasing function. Furthermore, by the inverse theorem function, it follows that $\frac{\mathrm{d}A}{\mathrm{d}\Psi} > 0$ and thus A is a one-to-one function of Ψ with an increasing inverse function $\Psi = \Psi(A)$

Case 2: $A < 0, -1 < \Psi < 0$.

From (2) one gets

$$\operatorname{sign}\left(\frac{\mathrm{d}\Psi}{\mathrm{d}A}\right) = \operatorname{sign}\,(2\sqrt{A\Psi} - A) \; = \operatorname{sign}\,(2\sqrt{-\Psi} + \sqrt{-A}), \tag{25}$$

and using (22) it follows that

$$2\sqrt{-\Psi} + \sqrt{-A} = \sqrt{-\Psi} + \frac{\arcsin(\sqrt{-\Psi})}{\sqrt{\Psi+1}} > 0. \tag{26}$$

Hence $\frac{\mathrm{d}A}{\mathrm{d}\Psi} > 0$ and there exist $\Psi = \Psi(A)$ as an increasing function. In summary, the following result has been established:

Theorem 2.1. *The nonlinear volatility correction function Ψ, unique solution of (2) satisfies the following properties:*

(i) *Ψ is implicitly defined by*

$$A = \left(-\frac{\operatorname{arcsinh}\sqrt{(\Psi)}}{\sqrt{\Psi+1}} + \sqrt{\Psi}\right)^2, \quad \text{if } \Psi > 0,$$

$$A = -\left(\frac{\arcsin\sqrt{(-\Psi)}}{\sqrt{\Psi+1}} - \sqrt{-\Psi}\right)^2, \quad \text{if } -1 < \Psi < 0.$$

(ii) *Ψ is a one to one increasing function mapping the real line onto the interval* $]-1, +\infty[$.

3. The semidicretization method: The linear B-S model

In this section we address the linear B-S model, i.e., the model (6)–(8), where $a = 0$ and $\sigma = \sigma_0$ in (1). Although the method will be used in the next section for the nonlinear case, we check the method here for the linear case because in this case the exact solution is known. Note that all the values u_{-1}, u_0, u_N and u_{N+1} appearing in (13) must be specified because they correspond to approximations of the numerical solution at the values of the underlying variable

$$S_{-1} = E - L - h, \qquad S_0 = E - L, \qquad S_N = E + L, \qquad S_{N+1} = E + L + h, \tag{27}$$

respectively. Unlike to the approach of [1] where the boundary conditions (8) are translated to the boundary of the bounded discretization domain, here using the semidiscretization technique we do not need the boundary values but they are obtained by interpolating the values of the numerical solution at the neighbours mesh internal nodes. Here, we use fourth-order Lagrange interpolating polynomial according with the order of approximation of scheme (9) and (10). Thus we have

$$\begin{aligned} u_{-1} &= 10u_1 - 20u_2 + 15u_3 - 4u_4, \\ u_0 &= 4u_1 - 6u_2 + 4u_3 - u_4\,, \\ u_N &= 4u_{N-1} - 6u_{N-2} + 4u_{N-3} - u_{N-4}\,, \\ u_{N+1} &= 10u_{N-1} - 20u_{N-2} + 15u_{N-3} - 4u_{N-4}. \end{aligned} \tag{28}$$

Taking into account (11)-(13) and (28), the semidiscretized problem takes the form

$$\frac{\mathrm{d}u}{\mathrm{d}\tau} = Au, \qquad u(0) = \left[u_1(0), \ldots, u_{N-1}(0)\right]^T, \tag{29}$$

where

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & 0 & \ldots & \ldots & 0 \\ a_{21} & a_{22} & a_{23} & a_{24} & 0 & \ddots & \ddots & \vdots \\ \alpha_3 & \beta_3 & \gamma_3 & \delta_3 & \xi_3 & 0 & \ddots & \vdots \\ 0 & \alpha_4 & \beta_4 & \gamma_4 & \delta_4 & \xi_4 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & 0 \\ \vdots & \ldots & 0 & \alpha_{N-3} & \beta_{N-3} & \gamma_{N-3} & \delta_{N-3} & \xi_{N-3} \\ \vdots & \ldots & \ldots & 0 & a_{N-2N-4} & a_{N-2N-3} & a_{N-2N-2} & a_{N-2N-1} \\ 0 & \ldots & \ldots & 0 & a_{N-1N-4} & a_{N-1N-3} & a_{N-1N-2} & a_{N-1N-1} \end{bmatrix}, \tag{30}$$

$$u_i(0) = \max(S_i - E, 0), \quad 1 \le i \le N-1, \tag{31}$$

and the entries of A are given by (14) taking $\sigma_i = \sigma_0$ for all i, $1 \leqslant i \leqslant N-1$, and the following expressions for the nonzero entries of the first, second, last but one and last rows

$$\left.\begin{array}{cc} a_{11} = \gamma_1 + 10\alpha_1 + 4\beta_1 & a_{12} = \delta_1 - 20\alpha_1 - 6\beta_1 \\ a_{13} = \xi_1 + 15\alpha_1 + 4\beta_1 & a_{14} = -4\alpha_1 - \beta_1 \\ a_{21} = \beta_2 + 4\alpha_2 & a_{22} = \gamma_2 - 6\alpha_2 \\ a_{23} = \delta_2 + 4\alpha_2 & a_{24} = \xi_2 - \alpha_2 \\ a_{N-2N-4} = \alpha_{N-2} - \xi_{N-2} & a_{N-2N-3} = \beta_{N-2} + 4\xi_{N-2} \\ a_{N-2N-2} = \gamma_{N-2} - 6\xi_{N-2} & a_{N-2N-1} = \delta_{N-2} + 4\xi_{N-2} \\ a_{N-1N-4} = -\delta_{N-1} - 4\xi_{N-1} & a_{N-1N-3} = \alpha_{N-1} + 4\delta_{N-1} + 15\xi_{N-1} \\ a_{N-1N-2} = \beta_{N-1} - 6\delta_{N-1} - 20\xi_{N-1} & a_{N-1N-1} = \gamma_{N-1} + 4\delta_{N-1} + 10\xi_{N-1}. \end{array}\right\} \tag{32}$$

The solution of (29) is given by

$$u(\tau) = \mathrm{e}^{A\tau} u(0). \tag{33}$$

Note that (32) has the advantage that it can be computed efficiently using MATLAB package.

The next example compares the value of the exact solution for the case of a call option pricing with the numerical values obtained using (32) for several values of $h = \frac{2L}{N}$.

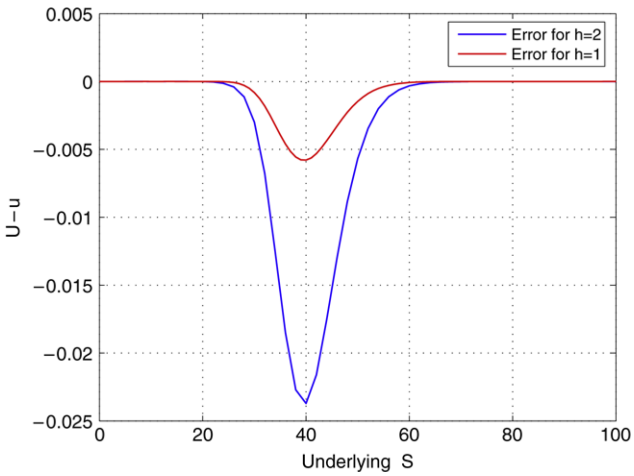

Fig. 1. Error of numerical solution for the linear case.

Example 1. Consider the vanilla call option with parameter

$$\sigma = 0.2, \qquad r = 0.04, \qquad E = 40, \qquad \tau = 0.5 \text{ years}.$$

Fig. 1 shows how the difference between the exact solution and the numerical solution with the semidiscretization technique at $t = 0$ decreases as h decreases.

4. The nonlinear case

In this section we consider the fourth-order semidiscretization of the nonlinear model (6)–(8) where σ is given by (1). The treatment of the numerical approximations u_{-1}, u_0, u_N *and* u_{N-1} are interpolated as in the linear case studied in Section 3. Hence, the semidiscretized problem takes the form

$$\frac{\mathrm{d}u}{\mathrm{d}\tau} = M(\tau)u(\tau), \quad 0 < \tau \leq T, \tag{34}$$

together with the initial condition (29) and (31), where $M(\tau)$ is given by

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & 0 & \dots & \dots & 0 \\ a_{21} & a_{22} & a_{23} & a_{24} & 0 & \ddots & \ddots & \vdots \\ \alpha_3 & \beta_3 & \gamma_3 & \delta_3 & \xi_3 & 0 & \ddots & \vdots \\ 0 & \alpha_4 & \beta_4 & \gamma_4 & \delta_4 & \xi_4 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & 0 \\ \vdots & \dots & 0 & \alpha_{N-3} & \beta_{N-3} & \gamma_{N-3} & \delta_{N-3} & \xi_{N-3} \\ \vdots & \dots & \dots & 0 & a_{N-2N-4} & a_{N-2N-3} & a_{N-2N-2} & a_{N-2N-1} \\ 0 & \dots & \dots & 0 & a_{N-1N-4} & a_{N-1N-3} & a_{N-1N-2} & a_{N-1N-1} \end{bmatrix}, \tag{35}$$

where entries values are given by (14) and (32) but now are nonconstant and involve numerical approximations of U_{SS}, see [1]. It is important to remark that we consider fully nonlinear problem without any linearization process like the authors of [1] where the nonlinear coefficient σ_i is frozen at each discretized τ level.

Using Euler method the numerical solution of (33) and (34) takes the form

$$u(\tau) = \left[\prod_{m=l-1}^{m=0} (I + kM(mk)) \right] u(0), \tag{36}$$

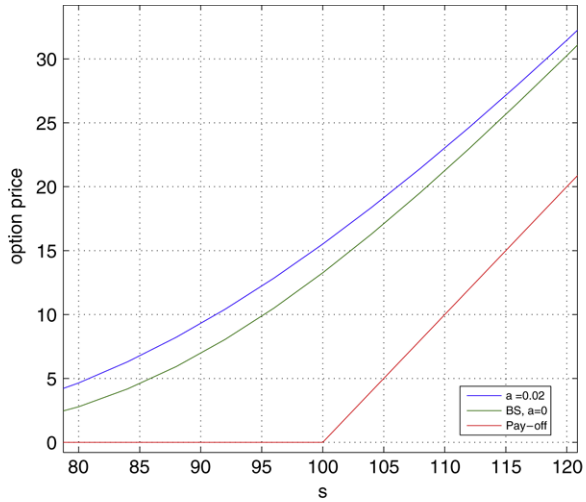

Fig. 2. Valuation of a vanilla call option in both linear and nonlinear cases.

where $k = \Delta\tau,\ lk = \tau,\ M(\tau)$ is given by (14) and (34) and

$$\sigma_i^2 = \sigma_0^2(1 + \Psi_i(\tau)), \tag{37}$$

and

$$\Psi_i(mk) = \Psi\left(e^{mkr} a^2 S_i^2 \left(\frac{-u_{i-2} + 16u_{i-1} - 30u_i + 16u_{i+1} - u_{i+2}}{12h^2}\right)\right), \tag{38}$$

being u_j evaluated at $\tau = mk$, see (1) and (10). Note that instead of approximating Ψ by solving numerically (2) we use the implicit expression given by Theorem 2.1.

The following example shows the numerical solutions obtained using the semidiscretization technique and formula (35)–(37). It can be checked that the numerical results obtained for the nonlinear case are practically coincident with those obtained in [1].

Example 2. Consider the vanilla call option with transaction costs and parameters

$$\sigma = 0.2, \qquad r = 0.1, \qquad E = 100, \qquad \tau = 1\text{year}, \qquad a = 0.02.$$

Fig. 2 shows option pricing valuation of this call option for the linear case and the nonlinear case with $a = 0.02$ as well as the pay-off function.

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